Product Rule and Permutation

Q:→ In how many ways can 3 people be seated in a row containing 7 seats?

sol:→ First person can be seated in 7 ways
2nd person " " " " 6 ways
3rd person " " " " 5 ways

By the fundamental principle of counting or Product rule, total no. of ways in which three persons can be seated in seven seats in a row

$= 7 \times 6 \times 5$

$= 210$

Q:→ How many 3-digits numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming that
(i) repetition of the digits is allowed?
(ii) repetition of the digits is not allowed?

sol:→
(i) Repetition allowed

$\underline{5} \times \underline{5} \times \underline{5} = 125$

(ii) Repetition not allowed

$\underline{3} \times \underline{4} \times \underline{5} = 60$

Q:→ Find the number of different signals that can be made by arranging at least three flags in order on a vertical pole, if 6 different flags are available.

Sol:→ No. of flags = 6

i) No. of signals with three flags:

$$\underline{4} \times \underline{5} \times \underline{6} = 120$$

ii) No. of signals with four flags

$$\underline{3} \times \underline{4} \times \underline{5} \times \underline{6} = 360$$

iii) No. of signals with five flags

$$\underline{2} \times \underline{3} \times \underline{4} \times \underline{5} \times \underline{6} = 720$$

iv) No. of signals with six flags

$$\underline{1} \times \underline{2} \times \underline{3} \times \underline{4} \times \underline{5} \times \underline{6} = 720$$

∴ Total no. of signals with atleast 3 flags

$$= 120 + 360 + 720 + 720 = 1920.$$

Practical Problems involving Permutation:

$${}^{n}P_{r} = \frac{\lfloor n}{\lfloor n-r}$$

Arrangement of r objects out of n distinct objects.

Q:→ How many 3-letter words can be made using the letters of the word ORIENTAL?

Sol:→ No. of letters in the word ORIENTAL = 8 = n

No. of letters to be taken at a time = 3 = r

$\therefore$ Reqd no. of 3 letters words $= {}^8P_3 = \frac{\lfloor 8}{\lfloor 5}$

$= 8 \times 7 \times 6$

$= 336$

Q:→ Find the no. of different 8 letter words formed from the letters of the word TRIANGLE if each word is to

i) begin with T.

ii) end with E

iii) begin with T and end with E

iv) have vowels occupying odd places.

sol:→ No. of letters in the word TRIANGLE = 8

No. of letters to be taken at a time = 8

(i) begin with T

$\underset{1}{T}\ \_\ \_\ \_\ \_\ \_\ \_\ \_$ 7 places 7 letters ${}^7P_7$

Reqd. no. of words $= 1 \times {}^7P_7 = {}^7P_7 = \lfloor 7 = 5040$

ii) End with E

$\_\ \_\ \_\ \_\ \_\ \_\ \_\ \overset{1}{E}$ ${}^7P_7$

7 places 7 letters

Reqd no. of words $= {}^7P_7 \times 1 = {}^7P_7 = \lfloor 7 = 5040$

iii) Begin with T and end with E

6 places 6 ways

I _ – – – – – – E

$1 \times {}^6P_6 \times 1 = \lfloor 6 = 720$

(iv) TRIANGLE

$$\underset{1}{X} \quad \underset{2}{\_} \quad \underset{3}{X} \quad \underset{4}{\_} \quad \underset{5}{X} \quad \underset{6}{\_} \quad \underset{7}{X} \quad \underset{8}{\_}$$

No. of odd positions $= 4$

No. of Vowels (I, A, E) $= 3$

The three vowels can be arranged at the 4 odd position in ${}^4P_3$ ways

and five consonants can be arranged in ${}^5P_5$ ways

Reqd no. of words $= {}^4P_3 \times {}^5P_5$

$= \lfloor 4 \times \lfloor 5$

$= 24 \times 120$

$= 2880.$

Q:→ How many words can be formed using letters of EQUATION so that vowels and consonants occur together?

Vowels E, U, A, I, O → 1 letter

Consonants Q, T, N → 1 letter

5 → ${}^5P_5$

3 → ${}^3P_3$

$${}^{2}P_{2} \times {}^{5}P_{5} \times {}^{3}P_{3}$$

$$\lfloor 2 \times \lfloor 5 \times \lfloor 3$$

$$2 \times 120 \times 6 = 1440$$ $\underline{\underline{\text{Ans}}}$